A direct-forcing immersed boundary projection method for simulating fluid-solid interaction problems

Suh-Yuh Yang (楊肅煜)

Department of Mathematics, National Central University
Jhongli District, Taoyuan City 32001, Taiwan

March 16, 2019

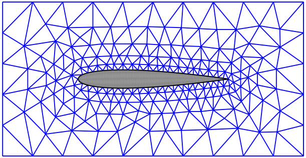
Taiwan-India Joint Conference@CASTS/NTU

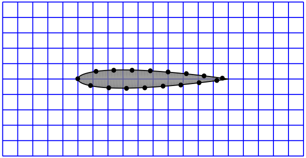
Outline of the talk

1. A direct-forcing immersed boundary projection method of Kajishima *et al.*
2. Inconsistency in the method
3. Two remedies to alleviate the inconsistency problem
4. Numerical experiments
5. Concluding remarks

Fluid-structure interaction problem

- The primary issues for CFD are accuracy, computational efficiency, and the ability to handle complex geometries.
- The fluid-structure interaction problem describes the coupling of fluid and structure mechanics. It usually requires the modeling of complex geometric structure and moving boundaries. Thus, it is very challenging for conventional body-fitted approach.





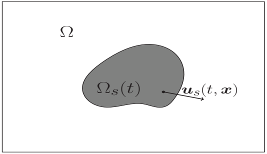
We will introduce a Cartesian-grid-based non-boundary conforming approach for fluid-solid interaction problems. More precisely, we will consider the direct-forcing immersed boundary projection method.

Fluid-solid interaction (FSI) problem

A simple one-way coupling FSI problem is flow over a stationary or moving solid body with a prescribed velocity.

Let Ω be the fluid domain which encloses a rigid body positioned at $\overline{\Omega}_s(t)$ *with a prescribed velocity* $\boldsymbol{u}_s(t, \boldsymbol{x})$. The FSI problem with initial value and no-slip boundary condition can be posed as follows:

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} - \nu \nabla^2 \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p &= \boldsymbol{f} \quad \text{in } (\Omega \setminus \overline{\Omega}_s) \times (0, T], \\
\nabla \cdot \boldsymbol{u} &= 0 \quad \text{in } (\Omega \setminus \overline{\Omega}_s) \times (0, T], \\
\boldsymbol{u} &= \boldsymbol{u}_b \quad \text{on } \partial\Omega \times [0, T], \\
\boldsymbol{u} &= \boldsymbol{u}_s \quad \text{on } \partial\Omega_s \times [0, T], \\
\boldsymbol{u} &= \boldsymbol{u}_0 \quad \text{in } (\Omega \setminus \overline{\Omega}_s) \times \{t = 0\},
\end{aligned}
$$



where $\boldsymbol{u}$ is the velocity field, p the pressure (divided by a constant density ρ), ν the kinematic viscosity, $\boldsymbol{f}$ the density of body force.

Direct-forcing immersed boundary (IB) approach

We first consider the solid object as a portion of the fluid and then introduce a virtual force $\boldsymbol{F}$ to the momentum equation, and we expect the problem can be solved on the whole domain Ω and do not need to set the interior boundary condition $\boldsymbol{u} = \boldsymbol{u}_s$ on the interface $\partial\Omega_s$:

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} - \nu \nabla^2 \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p &= \boldsymbol{f} + \boldsymbol{F} \quad \text{in } \Omega \times (0, T], \\
\nabla \cdot \boldsymbol{u} &= 0 \quad \text{in } \Omega \times (0, T], \\
\boldsymbol{u} &= \boldsymbol{u}_b \quad \text{on } \partial\Omega \times [0, T], \\
\boldsymbol{u} &= \boldsymbol{u}_0 \quad \text{in } \Omega \times \{t = 0\}.
\end{aligned}
$$

- *Note that the virtual force $\boldsymbol{F}$ is distributed only in the whole solid object region $\overline{\Omega}_s(t)$, making the region acts exactly as if it were a solid rigid body immersed in the fluid with a prescribed velocity $\boldsymbol{u}_s(t, \boldsymbol{x})$.*
- *But, at this moment, we do not know how to specify the virtual force $\boldsymbol{F}$ such that the region fulfills the prescribed velocity $\boldsymbol{u}_s(t, \boldsymbol{x})$.*

Time-discretization of the incompressible NS equations

Let us first discretize the time variable of the Navier-Stokes problem, with the spatial variable being left continuous. Consider the implicit Euler time-discretization with an explicit first-order approximation to the nonlinear convection. Then we have the BVP at time $t = t_{n+1}$:

$$\begin{aligned} \frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^n}{\Delta t} - \nu \nabla^2 \boldsymbol{u}^{n+1} + (\boldsymbol{u}^n \cdot \nabla)\boldsymbol{u}^n + \nabla p^{n+1} &= \boldsymbol{f}^{n+1} + \boldsymbol{F}^{n+1} \quad \text{in } \Omega, \\ \nabla \cdot \boldsymbol{u}^{n+1} &= 0 \quad \text{in } \Omega, \\ \boldsymbol{u}^{n+1} &= \boldsymbol{u}_b^{n+1} \quad \text{on } \partial\Omega. \end{aligned}$$

- *It is highly inefficient in solving this BVP directly, even if $\boldsymbol{F}^{n+1}$ is already known. This is the reason for proposing the projection approach to decouple the computation of $(\boldsymbol{u}^{n+1}, p^{n+1})$.*
- *Next, we will consider a direct-forcing IB approach based on the first-order Chorin projection scheme. The virtual force $\boldsymbol{F}^{n+1}$ will be specified in the scheme when we decouple the time-discretized problem.*

A direct-forcing IB projection method of Kajishima et al.

The main idea of the method was first proposed by Kajishima *et al.* (JSME-B 2001) & later by Noor-Chern-Horng (CM 2009).

***Step* 1 :** Solve for the intermediate velocity field $\boldsymbol{u}^*$,

$$\begin{cases} \dfrac{\boldsymbol{u}^* - \boldsymbol{u}^n}{\Delta t} - \nu \nabla^2 \boldsymbol{u}^* + (\boldsymbol{u}^n \cdot \nabla)\boldsymbol{u}^n = \boldsymbol{f}^{n+1} & \text{in } \Omega, \\ \boldsymbol{u}^* = \boldsymbol{u}_b^{n+1} & \text{on } \partial\Omega. \end{cases}$$

***Step* 2 :** Determine $\boldsymbol{u}^{**}$ and p^{n+1} by solving *(ensured by Helmholtz-Hodge D.)*

$$\begin{cases} \dfrac{\boldsymbol{u}^{**} - \boldsymbol{u}^*}{\Delta t} + \nabla p^{n+1} = \boldsymbol{0} & \text{in } \Omega, \\ \nabla \cdot \boldsymbol{u}^{**} = 0 & \text{in } \Omega, \\ \boldsymbol{u}^{**} \cdot \boldsymbol{n} = \boldsymbol{u}_b^{n+1} \cdot \boldsymbol{n} & \text{on } \partial\Omega. \end{cases}$$

It is equivalent to solving the Neumann Poisson problem:

$$\begin{cases} \nabla^2 p^{n+1} = \dfrac{1}{\Delta t} \nabla \cdot \boldsymbol{u}^* & \text{in } \Omega, \\ \nabla p^{n+1} \cdot \boldsymbol{n} = 0 & \text{on } \partial\Omega, \end{cases}$$

and set $\boldsymbol{u}^{**} = \boldsymbol{u}^* - \Delta t \nabla p^{n+1} \Longrightarrow \boldsymbol{u}^{**} \cdot \boldsymbol{n} = \boldsymbol{u}_b^{n+1} \cdot \boldsymbol{n}$ *on* $\partial\Omega$.

Method of Kajishima et al. (cont′d)

***Step* 3 :** Define the virtual force $\boldsymbol{F}^{n+1}$ and then determine the velocity field $\boldsymbol{u}^{n+1}$ by setting

$$\frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^{**}}{\Delta t} = \boldsymbol{F}^{n+1} := \eta \frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \quad \text{in } \Omega,$$

where $\eta(\boldsymbol{x}, t_{n+1})$ is defined by

$$\eta(\boldsymbol{x}, t_{n+1}) = \begin{cases} 1 & \boldsymbol{x} \in \overline{\Omega}_s^{n+1}, \\ 0 & \boldsymbol{x} \notin \overline{\Omega}_s^{n+1}. \end{cases}$$

The virtual force $\boldsymbol{F}^{n+1}$ exists on the whole solid body and zero elsewhere. In other words, in this step, we simply set

$$\boldsymbol{u}^{n+1} = \begin{cases} \boldsymbol{u}^{**} & \text{in } \overline{\Omega} \setminus \overline{\Omega}_s^{n+1}, \\ \boldsymbol{u}_s & \text{in } \overline{\Omega}_s^{n+1}. \end{cases}$$

Both Kajishima and Chern-Horng groups have successfully employed this rather simple method to study many fluid-solid interaction problems.

But, there is still an inconsistency problem in the method!

Second-order time-discretization

Using the implicit second-order Crank-Nicolson formula, we have

$$\begin{aligned} \frac{\boldsymbol{u}^{n+1}-\boldsymbol{u}^n}{\Delta t} - \frac{\nu}{2}\nabla^2(\boldsymbol{u}^{n+1}+\boldsymbol{u}^n) \\ +[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]^{n+\frac{1}{2}} + [\nabla p]^{n+\frac{1}{2}} &= [\boldsymbol{f}]^{n+\frac{1}{2}} + [\boldsymbol{F}]^{n+\frac{1}{2}} \quad \text{in } \Omega, \\ \nabla\cdot\boldsymbol{u}^{n+1} &= 0 \quad \text{in } \Omega, \\ \boldsymbol{u}^{n+1} &= \boldsymbol{u}_b^{n+1} \quad \text{on } \partial\Omega, \end{aligned}$$

where the notation $[g]^{n+\frac{1}{2}}$ denotes some second-order approximation to $\frac{1}{2}(g^{n+1}+g^n)$ or denotes the exact value. Two popular choices are:

$$\frac{1}{2}\left(g^{n+1}+g^n\right) = \frac{3}{2}g^n - \frac{1}{2}g^{n-1} + O(\Delta t^2) \qquad \text{(Adams-Bashforth)}$$

$$\frac{1}{2}\left(g^{n+1}+g^n\right) = g^{n+\frac{1}{2}} + O(\Delta t^2) \qquad \text{(midpoint)}$$

Again, it is inefficient to solve the semi-implicit equations directly and $[\boldsymbol{F}]^{n+\frac{1}{2}}$ is unknown. We will solve it by using the projection approach.

Direct-forcing IB method based on Brown et al. projection

***Step* 1 :** Solve for the intermediate velocity field $\boldsymbol{u}^*$,

$$\begin{cases} \dfrac{\boldsymbol{u}^* - \boldsymbol{u}^n}{\Delta t} - \dfrac{\nu}{2}\nabla^2(\boldsymbol{u}^* + \boldsymbol{u}^n) + [(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]^{n+\frac{1}{2}} + \nabla p^{n-\frac{1}{2}} = [\boldsymbol{f}]^{n+\frac{1}{2}} \text{ in } \Omega, \\ \boldsymbol{u}^* = \boldsymbol{u}_b^{n+1} \text{ on } \partial\Omega. \end{cases}$$

***Step* 2 :** Determine $\boldsymbol{u}^{**}$ and φ^{n+1} by solving

$$\begin{cases} \dfrac{\boldsymbol{u}^{**} - \boldsymbol{u}^*}{\Delta t} + \nabla\varphi^{n+1} = \boldsymbol{0} \quad \text{in } \Omega, \\ \nabla\cdot\boldsymbol{u}^{**} = 0 \quad \text{in } \Omega, \\ \boldsymbol{u}^{**}\cdot\boldsymbol{n} = \boldsymbol{u}_b^{n+1}\cdot\boldsymbol{n} \quad \text{on } \partial\Omega. \end{cases}$$

***Step* 3 :** Update the pressure by $p^{n+\frac{1}{2}} = p^{n-\frac{1}{2}} + \varphi^{n+1} - \dfrac{\nu\Delta t}{2}\nabla^2\varphi^{n+1}$.

***Step* 4 :** Define virtual force $\boldsymbol{F}^{n+\frac{1}{2}}$ and then determine velocity field $\boldsymbol{u}^{n+1}$ by setting

$$\frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^{**}}{\Delta t} = \boldsymbol{F}^{n+\frac{1}{2}} := \eta\frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \quad \text{in } \Omega.$$

Inconsistency in Kajishima's method

- Although the Kajishima method can produce reasonable results, it is not always convergent when the direct-forcing approach combined with an arbitrary chosen projection scheme such as the 2nd-order in time of Brown *et al.*, *unless the time step Δt is taken to be very small.*
- Note that at the new time level $t = t_{n+2}$, we have to solve

$$\frac{\boldsymbol{u}^* - \boldsymbol{u}^{n+1}}{\Delta t} - \frac{\nu}{2}\nabla^2(\boldsymbol{u}^* + \boldsymbol{u}^{n+1}) + [(\boldsymbol{u} \cdot \nabla)\boldsymbol{u}]^{n+\frac{3}{2}} + \nabla p^{n+\frac{1}{2}} = [\boldsymbol{f}]^{n+\frac{3}{2}}.$$

  But $\boldsymbol{u}^{n+1}$ & $p^{n+\frac{1}{2}}$ obtained in the previous step may be not consistent! Because at time t_{n+1}, we first determine $\boldsymbol{u}^{**}$ and $p^{n+\frac{1}{2}}$ simultaneously, and then we enforce $\boldsymbol{u}^{n+1} = \boldsymbol{u}^{**}$ in $\overline{\Omega} \setminus \overline{\Omega}_s^{n+1}$ & $\boldsymbol{u}^{n+1} = \boldsymbol{u}_s$ in $\overline{\Omega}_s^{n+1}$, but $p^{n+\frac{1}{2}}$ unchanged. That is, $p^{n+\frac{1}{2}}$ is consistent with $\boldsymbol{u}^{**}$, not $\boldsymbol{u}^{n+1}$.
- How to alleviate the inconsistency problem?
  We will use the idea of the prediction-correction approach and carefully choose a good projection scheme.

A direct-forcing IB projection method with PC (Choi-Moin)

Prediction stage:

Step P1 **:** Solve for the intermediate velocity field $\boldsymbol{u}^*$,

$$
\begin{cases}
\dfrac{\widetilde{\boldsymbol{u}} - \boldsymbol{u}^n}{\Delta t} - \dfrac{\nu}{2}\nabla^2(\widetilde{\boldsymbol{u}} + \boldsymbol{u}^n) + [(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]^{n+\frac{1}{2}} + \nabla p^{n-\frac{1}{2}} = [\boldsymbol{f}]^{n+\frac{1}{2}} & \text{in } \Omega, \\
\widetilde{\boldsymbol{u}} = \boldsymbol{u}_b^{n+1} & \text{on } \partial\Omega;
\end{cases}
$$

$$
\Longrightarrow \frac{\boldsymbol{u}^* - \widetilde{\boldsymbol{u}}}{\Delta t} - \nabla p^{n-\frac{1}{2}} = \boldsymbol{0} \quad \text{in } \Omega.
$$

Step P2 **:** Determine $\boldsymbol{u}^{**}$ and φ^{n+1} by solving

$$
\begin{cases}
\dfrac{\boldsymbol{u}^{**} - \boldsymbol{u}^*}{\Delta t} + \nabla\varphi^{n+1} = \boldsymbol{0} & \text{in } \Omega, \\
\nabla\cdot\boldsymbol{u}^{**} = 0 & \text{in } \Omega, \\
\boldsymbol{u}^{**}\cdot\boldsymbol{n} = \boldsymbol{u}^*\cdot\boldsymbol{n} & \text{on } \partial\Omega.
\end{cases}
$$

Step P3 **:** Predict the virtual force $\widetilde{\boldsymbol{F}}^{n+\frac{1}{2}}$ by setting

$$
\frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^{**}}{\Delta t} = \widetilde{\boldsymbol{F}}^{n+\frac{1}{2}} := \eta\,\frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \quad \text{in } \Omega.
$$

A direct-forcing IB projection method with PC (Choi-Moin)

Correction stage:

Step C1 **:** Solve for the intermediate velocity field $\boldsymbol{u}^*$,

$$\left\{ \begin{array}{r} \dfrac{\widetilde{\boldsymbol{u}} - \boldsymbol{u}^n}{\Delta t} - \dfrac{\nu}{2}\nabla^2(\widetilde{\boldsymbol{u}} + \boldsymbol{u}^n) + [(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]^{n+\frac{1}{2}} + \nabla p^{n-\frac{1}{2}} = [\boldsymbol{f}]^{n+\frac{1}{2}} + \widetilde{\boldsymbol{F}}^{n+\frac{1}{2}} \text{ in } \Omega, \\ \widetilde{\boldsymbol{u}} = \boldsymbol{u}_b^{n+1} \text{ on } \partial\Omega; \end{array} \right.$$

$$\Longrightarrow \frac{\boldsymbol{u}^* - \widetilde{\boldsymbol{u}}}{\Delta t} - \nabla p^{n-\frac{1}{2}} = \boldsymbol{0} \quad \text{in } \Omega.$$

Step C2 **:** Determine $\boldsymbol{u}^{**}$ and correct φ^{n+1} by solving

$$\left\{ \begin{array}{rcll} \dfrac{\boldsymbol{u}^{**} - \boldsymbol{u}^*}{\Delta t} + \nabla\varphi^{n+1} & = & \boldsymbol{0} & \text{in } \Omega, \\ \nabla\cdot\boldsymbol{u}^{**} & = & 0 & \text{in } \Omega, \\ \boldsymbol{u}^{**}\cdot\boldsymbol{n} & = & \boldsymbol{u}^*\cdot\boldsymbol{n} & \text{on } \partial\Omega. \end{array} \right.$$

Step C3 **:** Update the pressure as $p^{n+\frac{1}{2}} = \varphi^{n+1} - \dfrac{\nu}{2}\nabla\cdot\widetilde{\boldsymbol{u}}$.

Step C4 **:** Correct the velocity $\boldsymbol{u}^{n+1}$ and virtual force $\boldsymbol{F}^{n+\frac{1}{2}}$,

$$\frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^{**}}{\Delta t} = \eta\frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \text{ in } \Omega, \quad \boldsymbol{F}^{n+\frac{1}{2}} := \widetilde{\boldsymbol{F}}^{n+\frac{1}{2}} + \eta\frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \text{ in } \overline{\Omega}_s^{n+1}.$$

Second remedy for the inconsistency

Step **1 :** Solve for the intermediate velocity field $\boldsymbol{u}^*$,

$$\left\{ \begin{aligned} \frac{\widetilde{\boldsymbol{u}} - \boldsymbol{u}^n}{\Delta t} - \frac{\nu}{2}\nabla^2(\widetilde{\boldsymbol{u}} + \boldsymbol{u}^n) + [(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}]^{n+\frac{1}{2}} + \nabla p^{n-\frac{1}{2}} &= [\boldsymbol{f}]^{n+\frac{1}{2}} + \boldsymbol{F}^{n-\frac{1}{2}} \text{ in } \Omega, \\ \widetilde{\boldsymbol{u}} &= \boldsymbol{u}_b^{n+1} \text{ on } \partial\Omega; \end{aligned} \right.$$

$$\Longrightarrow \frac{\boldsymbol{u}^* - \widetilde{\boldsymbol{u}}}{\Delta t} - \nabla p^{n-\frac{1}{2}} = \boldsymbol{0} \quad \text{in } \Omega.$$

Step **2 :** Determine $\boldsymbol{u}^{**}$ and correct φ^{n+1} by solving

$$\left\{ \begin{aligned} \frac{\boldsymbol{u}^{**} - \boldsymbol{u}^*}{\Delta t} + \nabla\varphi^{n+1} &= \boldsymbol{0} \quad \text{in } \Omega, \\ \nabla\cdot\boldsymbol{u}^{**} &= 0 \quad \text{in } \Omega, \\ \boldsymbol{u}^{**}\cdot\boldsymbol{n} &= \boldsymbol{u}^*\cdot\boldsymbol{n} \quad \text{on } \partial\Omega. \end{aligned} \right.$$

Step **3 :** Update the pressure as $p^{n+\frac{1}{2}} = \varphi^{n+1} - \frac{\nu}{2}\nabla\cdot\widetilde{\boldsymbol{u}}$.

Step **4 :** Determine velocity $\boldsymbol{u}^{n+1}$ and virtual force $\boldsymbol{F}^{n+\frac{1}{2}}$,

$$\frac{\boldsymbol{u}^{n+1} - \boldsymbol{u}^{**}}{\Delta t} = \eta\frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \text{ in } \Omega, \quad \boldsymbol{F}^{n+\frac{1}{2}} := \boldsymbol{F}^{n-\frac{1}{2}} + \eta\frac{\boldsymbol{u}_s - \boldsymbol{u}^{**}}{\Delta t} \text{ in } \overline{\Omega}_s^{n+1}.$$

Space-discretization on a staggered grid

In the following numerical examples, we will employ the prediction-correction direct-forcing IB projection method (based on 2nd-order Choi-Moin scheme) and apply the second-order centered differences over a staggered Cartesian grid for space-discretization in the projection scheme:

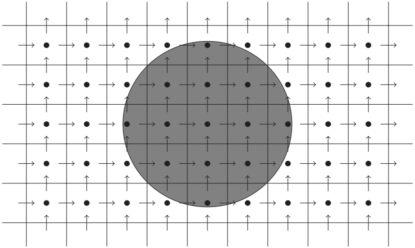
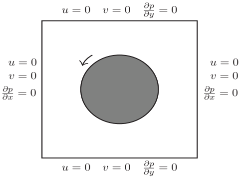


Diagram of the computational domain Ω with staggered grid, where the unknowns u, v and p are approximated at the grid points marked by $\rightarrow$, $\uparrow$ and $\bullet$, respectively

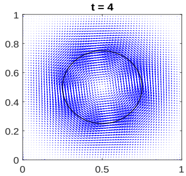
Rotating solid disk

Problem setting:

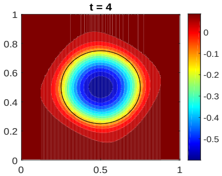
- The computational domain is $\Omega = (0,1) \times (0,1)$, within which there is a rotating solid disk centered at $(0.5, 0.5)$ with radius 0.25. The disk rotates counterclockwise by a constant angular velocity $\omega = 4$.
- The Reynolds number is $Re := 1/\nu = 100$, time step length is $\Delta t = 0.1h$ (CFL number $= 0.1$), and $T = 4$.



boundary conditions



velocity field



pressure contours

Error behavior

Error behavior of the numerical solutions u_h, v_h, and p_h at $T = 4$ using the solution of $h = 1/1620$ as the reference solution

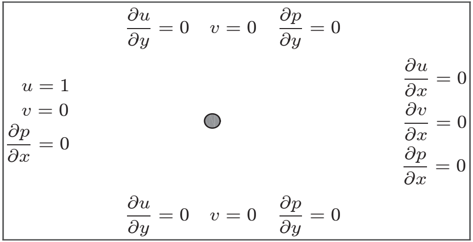
	$1/h$	1-norm	order	2-norm	order	max-norm	order
u_h	20	2.0820e-02	–	3.9742e-02	–	1.7573e-01	–
	60	8.4854e-03	0.82	1.7044e-02	0.77	8.2900e-02	0.68
	180	2.5123e-03	1.11	5.0608e-03	1.11	2.8370e-02	0.98
	540	6.5240e-04	1.23	1.3207e-03	1.22	8.1061e-03	1.14
v_h	20	2.5334e-02	–	4.2845e-02	–	1.7573e-01	–
	60	1.0199e-02	0.83	1.8496e-02	0.76	8.2900e-02	0.68
	180	3.0741e-03	1.09	5.5503e-03	1.10	2.8554e-02	0.97
	540	7.9659e-04	1.23	1.4500e-03	1.22	8.1061e-03	1.15
p_h	20	6.8326e-03	–	1.3968e-02	–	8.4475e-02	–
	60	3.0749e-03	0.73	6.2523e-03	0.73	4.8072e-02	0.51
	180	9.8066e-04	1.04	2.1771e-03	0.96	3.8831e-02	0.19
	540	2.6861e-04	1.18	7.5445e-04	0.96	2.5701e-02	0.38

The convergence rate of velocity seems to be super-linear in 1-norm, 2-norm and max-norm!

Flow past a stationary cylinder

Problem setting:

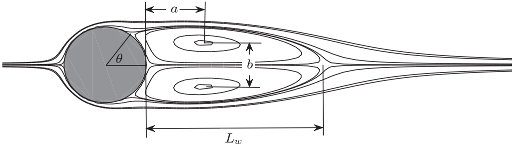
- $\Omega = (-13.4D, 16.5D) \times (-8.35D, 8.35D)$, where D is the diameter of the cylinder and we take $D = 0.2$.
- A non-uniform grid 250×160 is adopted to discretize the computational domain, within which a uniform grid 60×60 is employed in the region $[-D, D] \times [-D, D]$.
- The small uniform mesh size is $h = 2D/60$ and time step length is $\Delta t = 0.4h$ (*CFL* number is 0.4).



Numerical results at Re = 40

The comparison of experimental and numerical results of steady state wake dimensions and maximum drag coefficient for Re = 40

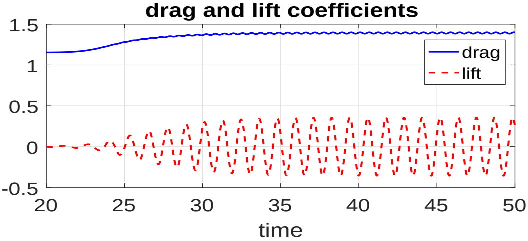
Methods	C_d	L_w/D	a/D	b/D	θ
Calhoun	1.62	2.18	–	–	54.2
Coutanceau-Bouard*	–	2.13	0.76	0.59	53.8
Linnick-Fasel	1.54	2.28	0.72	0.60	53.6
Su *et al.*	1.63	–	–	–	–
Taira-Colonius (B)	1.54	2.30	0.73	0.60	53.7
Tritton*	1.48	–	–	–	–
Ye *et al.*	1.52	2.27	–	–	–
Present method-PC	1.56	2.18	0.72	0.60	53.3



About Re ≤ 47, two symmetrical vortices will be stationarily attached behind the cylinder.

Drag and lift coefficients at Re=100

By increasing the value of *Re*, the symmetrical vortices will become unstable and break apart, leading to an alternating vortex shedding.



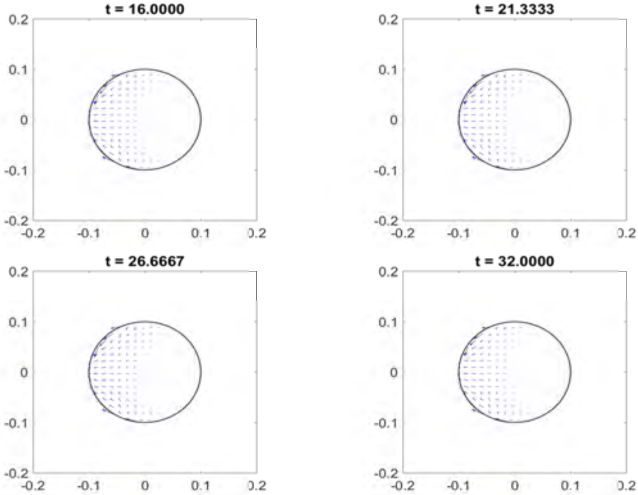
The drag and lift coefficients, C_d and C_ℓ, are defined as

$$C_d = \frac{F_d}{U_\infty^2 D/2} \quad \text{and} \quad C_\ell = \frac{F_\ell}{U_\infty^2 D/2},$$

where the drag and lift forces, F_d and F_ℓ, are calculated by

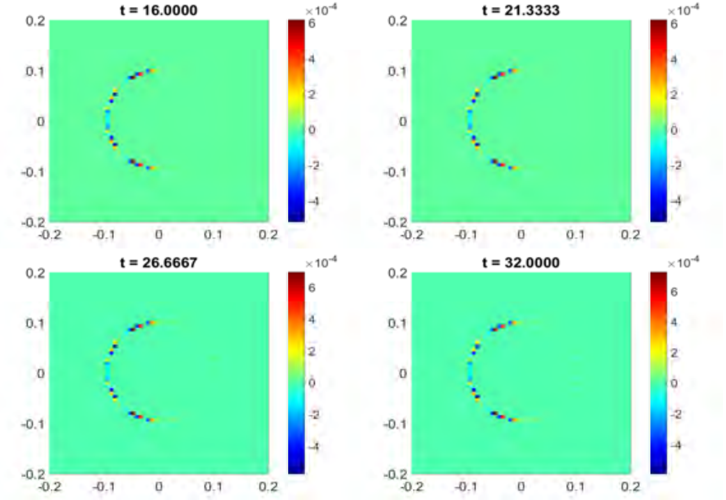
$$F_d = -\int_\Omega F_1\, d\boldsymbol{x} \approx -\sum_{\boldsymbol{x}_{ij}} F_1 h^2 \quad \text{and} \quad F_\ell = -\int_\Omega F_2\, d\boldsymbol{x} \approx -\sum_{\boldsymbol{x}_{ij}} F_2 h^2.$$

Instantaneous virtual force F distributed on $\overline{\Omega}_s$



The direct-forcing IB projection method with PC based on the Choi-Moin method

Instantaneous sink-source distribution : $\int_{\text{cell}} \nabla \cdot \boldsymbol{u}_h \, dxdy$



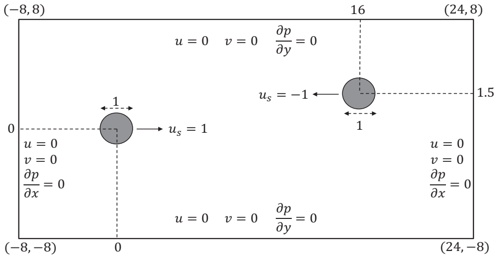
The direct-forcing IB projection method with PC based on the Choi-Moin method

Two cylinders moving towards each other

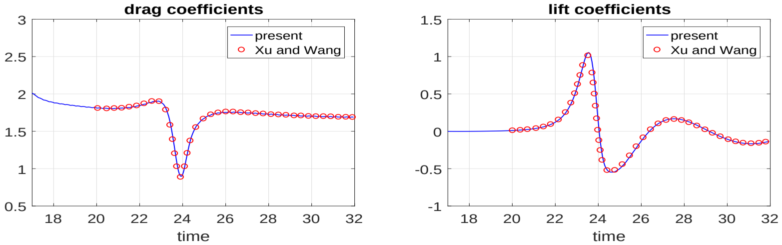
Problem setting:

- In this problem, $Re = 40$. A uniform grid 640×320 is adopted to discretize the computational domain $\Omega = (-8, 24) \times (-8, 8)$, and the time step is $\Delta t = 1/200$, CFL number $= 0.1$.
- The motion of the lower and upper cylinders are governed by setting the dynamics of their centers $(x_{\mathrm{lc}}, 0)$ and $(x_{\mathrm{uc}}, 1.5)$ to

$$x_{\mathrm{lc}} = \begin{cases} \frac{4}{\pi}\sin\left(\frac{\pi t}{4}\right), & 0 \le t \le 16, \\ t - 16, & 16 \le t \le 32, \end{cases} \qquad x_{\mathrm{uc}} = \begin{cases} 16 - \frac{4}{\pi}\sin\left(\frac{\pi t}{4}\right), & 0 \le t \le 16, \\ 32 - t, & 16 \le t \le 32. \end{cases}$$



Two cylinders moving towards each other



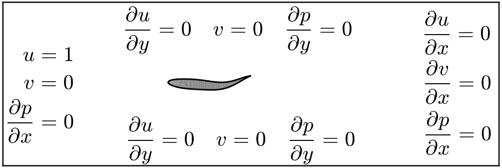
The time evolution of drag and lift coefficients, C_d and C_ℓ, for the upper cylinder compared with the results of Xu-Wang (JCP 2006, immersed interface method, $\Delta t = 1/2000$)

Our results are in good agreement with that of Xu-Wang (JCP 2006)

Flow past a swimming fish-like solid body

Problem setting: the fish motion is prescribed, i.e., $\overline{\Omega}_s(t)$ and $\boldsymbol{u}_s(t, \boldsymbol{x})$ are given. *(NACA0012 airfoil with the oscillation equation)*

- Reynolds number is defined as $Re = U_\infty L/\nu$, where L is the chord length of wavy foil. In this simulation, $L = U_\infty = 1$ and $Re = 5000$.
- Computational domain size is $6L \times 2L$, $\Omega = (-2, 4) \times (-1, 1)$.
- $\Delta x = \Delta y = 1/480$, $\Delta t = 0.0002$, CFL number ≈ 0.1, and the final time $T = 20$.



Please see some animations of the numerical simulations.

The two-way fluid-solid interaction problem

The fluid-solid interaction of the freely falling solid body with a virtual force can be formulated as the following initial-boundary value problem: *find* $\boldsymbol{u}$, p, $\boldsymbol{F}$, $\boldsymbol{u}_c$ *and* ω *with* $\int_\Omega p = 0$ *such that*

$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} - \nu \nabla^2 \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p = \boldsymbol{f} + \boldsymbol{F} \quad t \in (0, T], \quad \boldsymbol{x} \in \Omega, \\
\nabla \cdot \boldsymbol{u} = 0 \quad t \in (0, T], \quad \boldsymbol{x} \in \Omega, \\
\boldsymbol{u} = \boldsymbol{u}_b \quad t \in (0, T], \quad \boldsymbol{x} \in \partial\Omega, \\
\boldsymbol{u} = \boldsymbol{u}_0 \quad t = 0, \quad \boldsymbol{x} \in \overline{\Omega},
\end{aligned}
$$

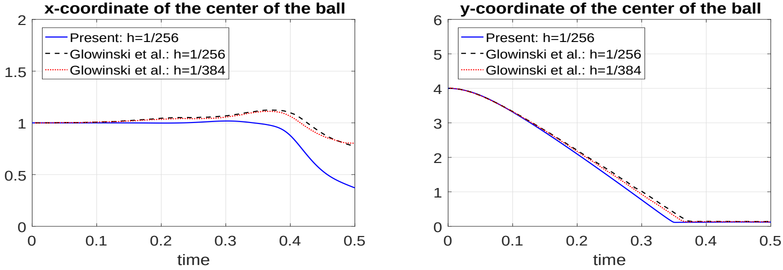
$$
\begin{aligned}
\boldsymbol{u} = \boldsymbol{u}_s := \boldsymbol{u}_c + \omega \times \boldsymbol{r} \quad \text{in } \overline{\Omega}_s, \\
(M_s - M_f)\frac{d\boldsymbol{u}_c}{dt} = (M_s - M_f)\boldsymbol{g} - \int_{\Omega_s} \rho_f \boldsymbol{F} dV, \quad \boldsymbol{u}_c(0) = \boldsymbol{u}_{c0}, \\
(\boldsymbol{I}_s - \boldsymbol{I}_f)\frac{d\omega}{dt} = -\int_{\Omega_s} \rho_f \boldsymbol{r} \times \boldsymbol{F} dV, \quad \omega(0) = \omega_0,
\end{aligned}
$$

where we consider a 2-D solid object of constant density ρ_s positioned at $\overline{\Omega}_s$ with centroid $\boldsymbol{X}_c$, translational velocity $\boldsymbol{u}_c$ and angular velocity ω.

Sedimentation of a circular body : $\nu = 0.01$

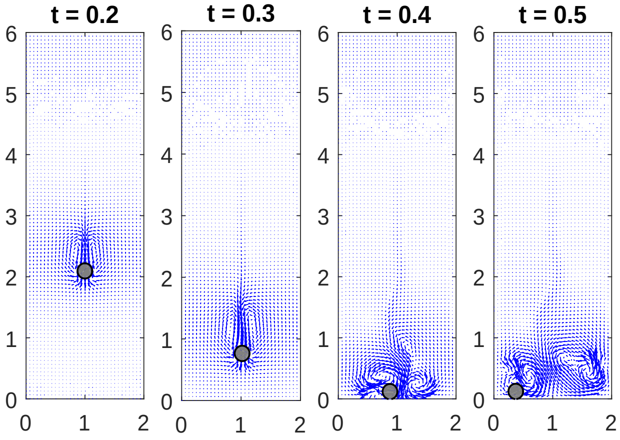
Problem setting:

- The computational domain $\Omega = (0,2) \times (0,6)$.
- The diameter of the body is $d = 0.25$ and is located at $(1,4)$ at time $t = 0$.
- The fluid density is $\rho_f = 1$ and the disk density $\rho_s = 1.5$.
- $\nu = 0.01, h = 1/256$, and $\Delta t = 7.5 \times 10^{-5}$.



Time evolution of position of the center of the ball

Flow field visualization



Please see some animations of the numerical simulations of freely falling solid bodies in an incompressible viscous fluid.

Thermal fluid-solid interaction problem

Let $\Omega \subset \mathbb{R}^2$ be a bounded fluid domain which encloses a single rigid solid body positioned at $\overline{\Omega}_s$. The direct-forcing approach to the thermal fluid-solid interaction problem can be posed as:

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} - P_1 \nabla^2 \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p &= \boldsymbol{f} + \boldsymbol{F} + P_2 \theta \boldsymbol{e}, \\ \nabla \cdot \boldsymbol{u} &= 0, \\ \frac{\partial \theta}{\partial t} - P_3 \nabla^2 \theta + \boldsymbol{u} \cdot \nabla \theta &= E, \end{aligned}$$

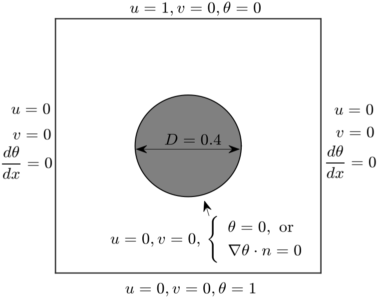
together with IC, BC on $\partial\Omega$, and internal BC on $\partial\Omega_s$:

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{u}_s \\ \theta &= \theta_s, \text{ or } -\kappa \frac{\partial \theta}{\partial \boldsymbol{n}} = Q_s. \end{aligned}$$

Here $\boldsymbol{F}$ and E are the momentum forcing and energy forcing, respectively. $\boldsymbol{e} = (0, 1)^\top$ is the unit vector in the vertical direction.

Mixed convection (with buoyancy)

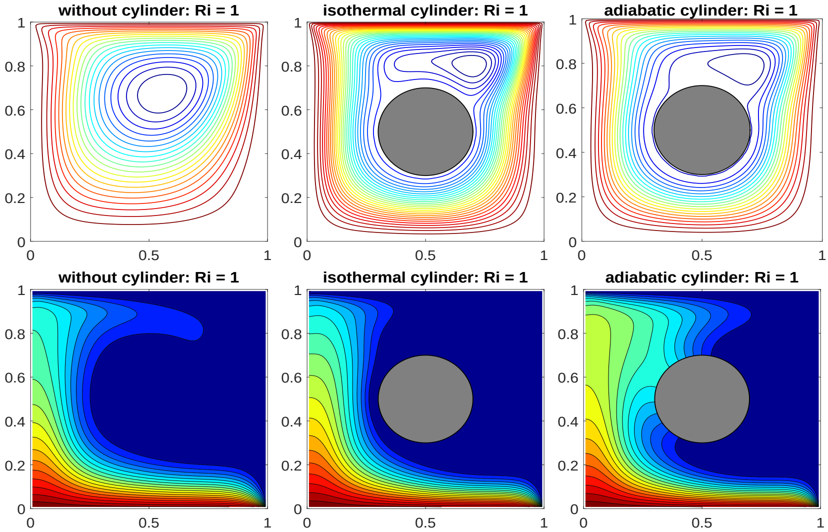
We consider a mixed convection problem in a lid-driven cavity with an embedded cylinder:



Reynolds number $Re = 100$, Prandtl number $Pr = 0.7$, Richardson number $Ri = \frac{Gr}{Re^2} = 0.01, 1, 5$, with Grashof number Gr,

$$P_1 = \frac{1}{Re} = \frac{1}{100}, \quad P_2 = \frac{Gr}{Re^2}, \quad P_3 = \frac{1}{RePr}.$$

Numerical results



Concluding remarks

1. We have developed a simple direct-forcing IB projection method for FSI problems, where the immersed rigid body can be stationary or moving in the fluid. This approach alleviates the inconsistency problem in the method of Kajishima *et al.*
2. Further works are still in progress for studying the dynamics of freely falling body in an incompressible viscous fluid and the thermal FSI problems.
3. Partial details about today's talk can be found in

   T.-L. Horng, P.-W. Hsieh, S.-Y. Yang, and C.-S. You,*
   A simple direct-forcing immersed boundary projection method with prediction-correction for fluid-solid interaction problems,
   Computers & Fluids, 176 (2018), pp. 135-152.

Concluding remarks

1. We have developed a simple direct-forcing IB projection method for FSI problems, where the immersed rigid body can be stationary or moving in the fluid. This approach alleviates the inconsistency problem in the method of Kajishima *et al.*
2. Further works are still in progress for studying the dynamics of freely falling body in an incompressible viscous fluid and the thermal FSI problems.
3. Partial details about today's talk can be found in

   T.-L. Horng, P.-W. Hsieh, S.-Y. Yang, and C.-S. You,*
   A simple direct-forcing immersed boundary projection method with prediction-correction for fluid-solid interaction problems,
   Computers & Fluids, 176 (2018), pp. 135-152.

Thank you for your attention!